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Resonant transmission and quantized charge transfer in adiabatic **quantum pumping**

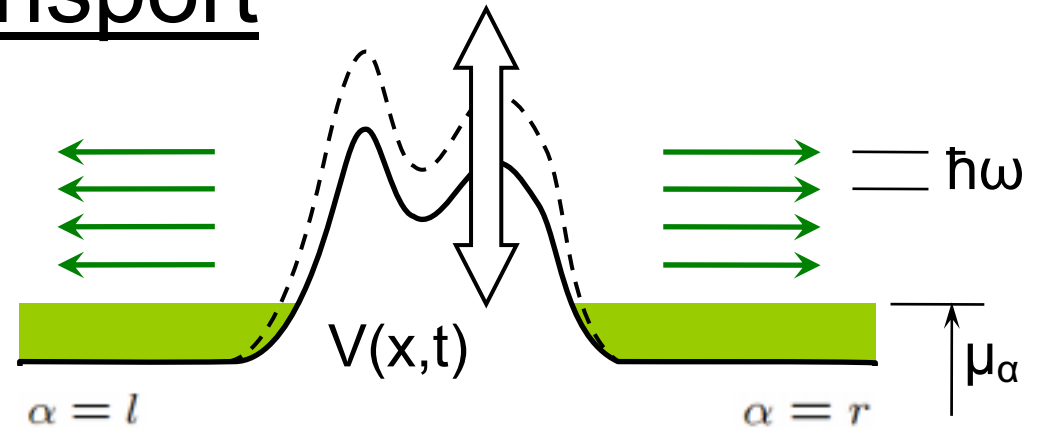
Phys. Rev. B **69**, 195301 (2004)

Eur. Phys. J B **39**, 385 (2004)

Theory of adiabatic transport

- Nanostructure connected to remote equilibrium reservoirs
- Coherent spinless electrons, no correlations
- External periodic potential acts locally:

$$V(x, t) = V(x, t + 2\pi/\omega)$$



- Consider t in $V(x, t)$ as a slow variable. Exact time-dependent scattering states are obtained from instantaneous (“frozen- t ”) energy eigenstates $|\chi_\alpha^t\rangle$ as a perturbation series in powers of $\partial/\partial t$ [1].
- The instantaneous current from lead α in the leading (adiabatic) order is a sum of

$$I_\alpha^{\text{pump}}(t) = \frac{e}{4\pi\hbar} \int dE \langle \chi_\alpha^t | \dot{V} | \chi_\alpha^t \rangle \frac{\partial(f_l + f_r)}{\partial E} \quad (\text{equivalent of Brouwer f-la [2]})$$

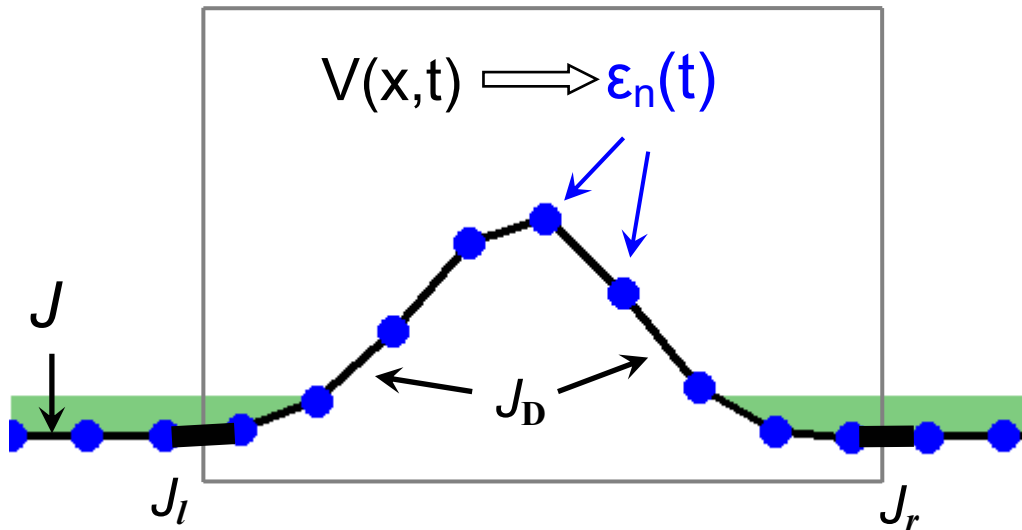
$$I_\alpha^{\text{bias}}(t) = \frac{e}{2\pi\hbar} \int dE \left\{ (f_l - f_r) \mathcal{T} + \frac{\hbar}{2} \frac{\partial(f_l - f_r)}{\partial E} \mathcal{T} \dot{\theta} \right\} \quad (\text{equivalent of Landauer f-la})$$

Here \mathcal{T} is the probability and θ is the phase of instantaneous transmission, $f(E)$ is Fermi distr.

- Adiabaticity holds as long as $\hbar\omega \ll$ typical energy scale of $\mathcal{T}(E)$ variation (e.g., resonance width Γ) [1,7]

Requires general solution of the static scattering problem at every t

Calculation method



- Discretize external potential on N points (tight-binding sites)
- On-site energies

$$\epsilon_n(t) \equiv V(na, t)$$

- Nearest-neighbour hoppings J describe the kinetic energy:

$$\mu = -2J \cos k_F a$$

Charge transferred per one period (zero temperature and zero-bias) [3]:

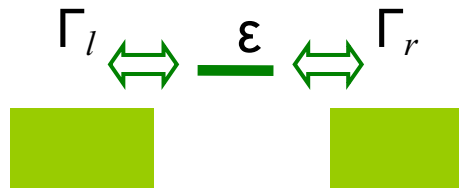
$$Q_\alpha \equiv \int_0^{2\pi/\omega} I_\alpha^{\text{pump}}(t) dt = \frac{e J_\alpha \sin k_F a}{\pi J} \int \sum_{n=1}^N |g_{n,1}|^2 \dot{\epsilon}_n dt$$

- Integrand is an inverse of a $N \times N$ matrix (Green's function for fixed t and E)
- If $V(x,t)$ is a polynomial in $\sin \omega t$ and $\cos \omega t$, the integration is done analytically

Gives the adiabatically pumped charge Q , once $V(x,t)$ is specified

Resonance approximation

A weakly coupled state shows a Breit-Wigner resonance in



transmission

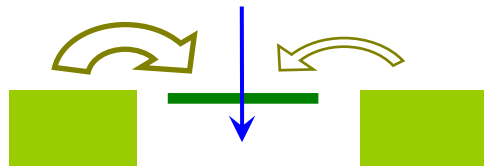
$$T = \frac{\Gamma_l \Gamma_r}{(\mu - \varepsilon)^2 + (\Gamma_l + \Gamma_r)^2/4}$$

pumping current [4]

$$I_\alpha = \frac{e}{2\pi} \frac{-\Gamma_\alpha \dot{\varepsilon} + \dot{\Gamma}_\alpha (\mu - \varepsilon)}{(\mu - \varepsilon)^2 + (\Gamma_l + \Gamma_r)^2/4}$$

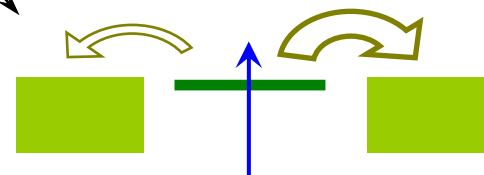
If the couplings vary slowly, the Lorentzian integrates to 1:

$$\Delta Q_\alpha \equiv \int_{t_1}^{t_2} I_\alpha dt \approx \pm \frac{\Gamma_\alpha}{\Gamma_l + \Gamma_r} \times e$$



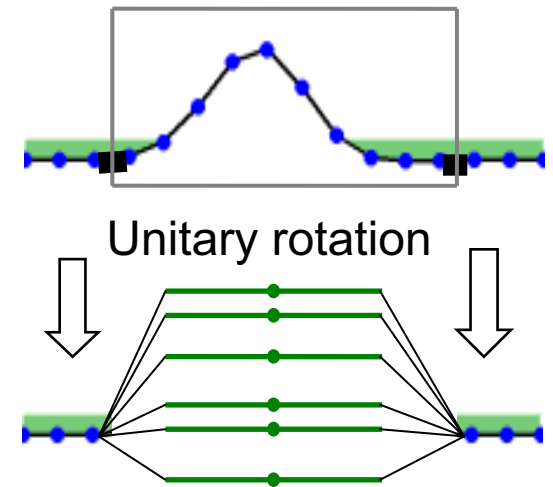
$$\Delta Q_\alpha > 0$$

Loading, $d\varepsilon/dt < 0$



$$\Delta Q_\alpha < 0$$

Unloading, $d\varepsilon/dt > 0$

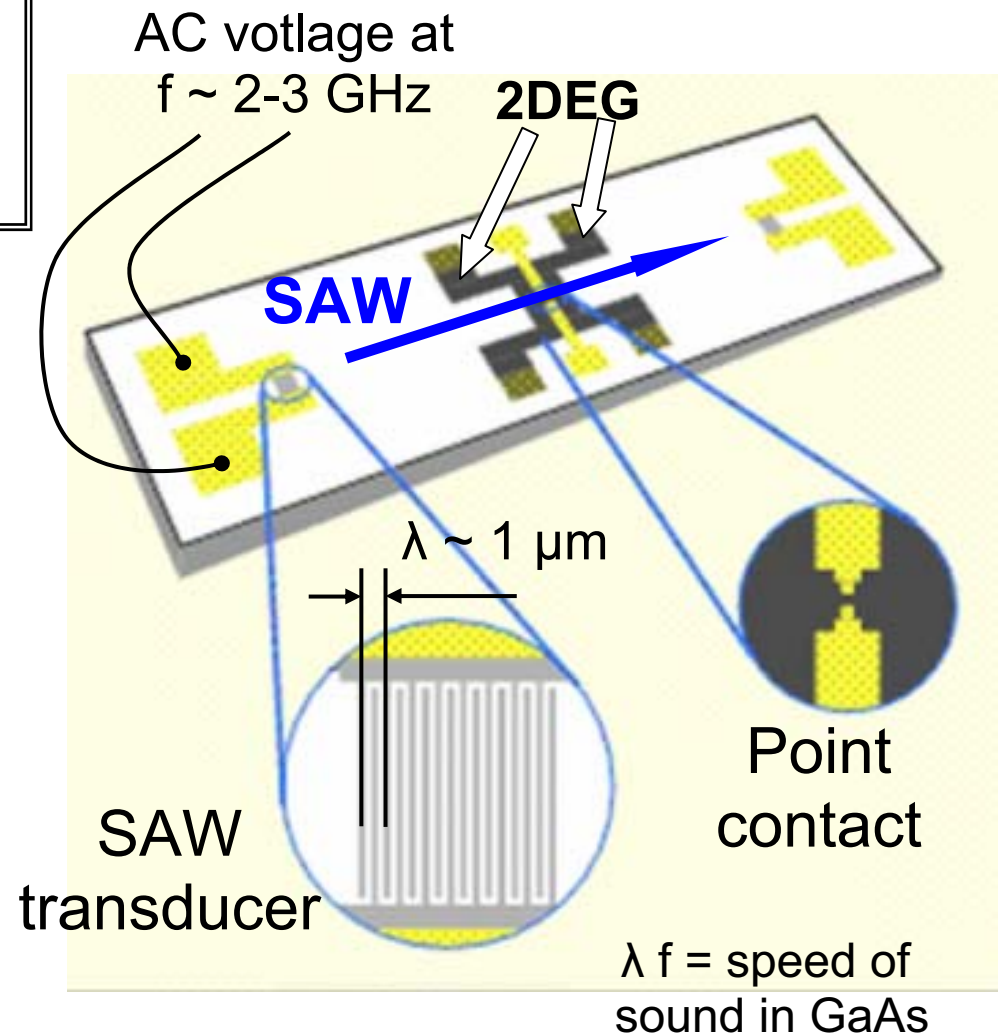
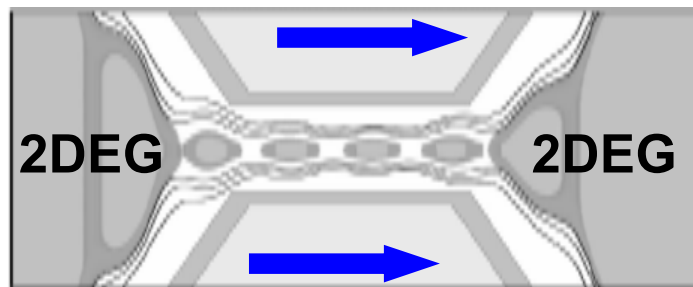


Can be generalized to several states

Approximates Q as a sum of loading/unloading contributions

Quantized pumping with Surface Acoustic Waves

- A running wave of mechanical deformation creates a **moving potential profile** due to piezoelectric properties of GaAs
- In the depleted region of a point contact **screening is reduced**
- Periodic potential can capture and transfer an integer number of electrons



- Quantized transport achieved in experiments by V.I.Talyanskii *et al.* (Cambridge, UK, 1996 – ...) [5]

What can we learn from an idealized adiabatic pumping model?

A simple model

- 1D geometry (single transverse mode)
- Assume complete screening outside a region of length L
- Take the simplest form for the potential induced by the gates and the SAW[3,6]:

The diagram illustrates the potential $V(x, t)$ and its components. The equation is
$$V(x, t) = \left[-V_g + P \cos(qx - \omega t) \right] \times e^{-x^2/L_s^2}$$
 Annotations include:

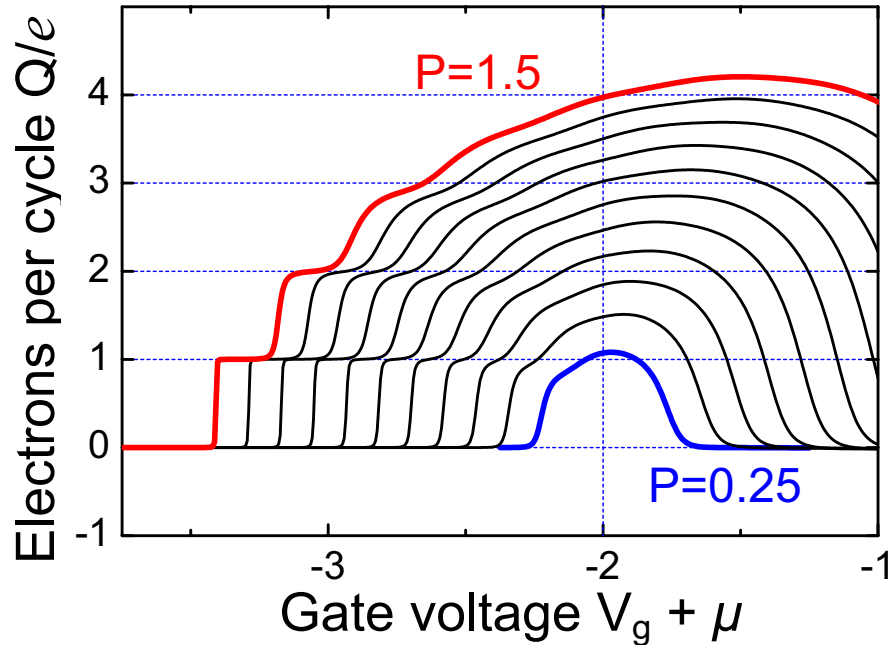
- A box labeled "Gate voltage defines a static barrier" with an arrow pointing to $-V_g$.
- A box labeled "Optional gradual screening" with an arrow pointing to the exponential term e^{-x^2/L_s^2} .
- A box labeled "SAW amplitude" with an arrow pointing to P .
- A box labeled "SAW wave-vector $q = 2\pi/\lambda$ " with an arrow pointing to q .

- Charge transfer Q per period (DC component of the acoustoelectric current) is calculated using the **theory of adiabatic pumping** [1,3] (see details on the left)

Calculation outcome

Main experimental features are reproduced:

- quantization of the acoustoelectirc current below conduction pinch-off
- greater SAW amplitude results in more steps
- higher steps are less accurate



Acoustoelectirc current =
 = frequency \times (1, 2, 3, ...) $\times e$

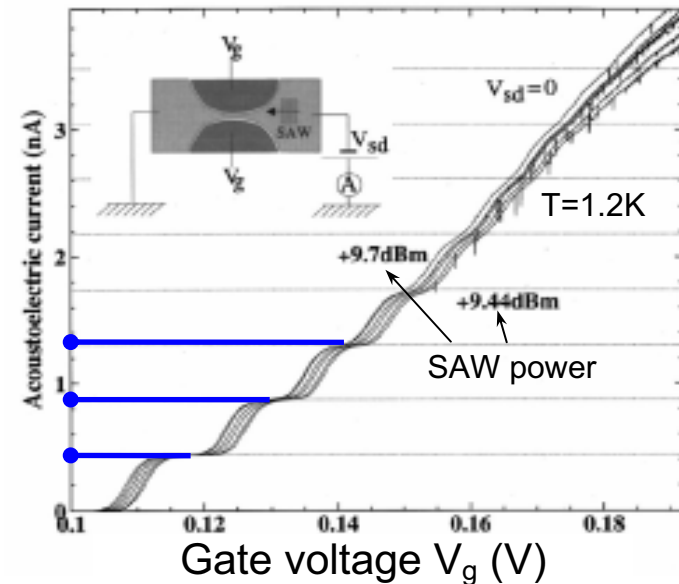
Model parameters:

$$\lambda = 2 L ; L_s = 4 L$$

$J = 1$ (sets the unit of energy)

$k_F a = \pi/12$ (like free electrons)

$N \equiv L / a = 24$ sites



Experimental data from [5]

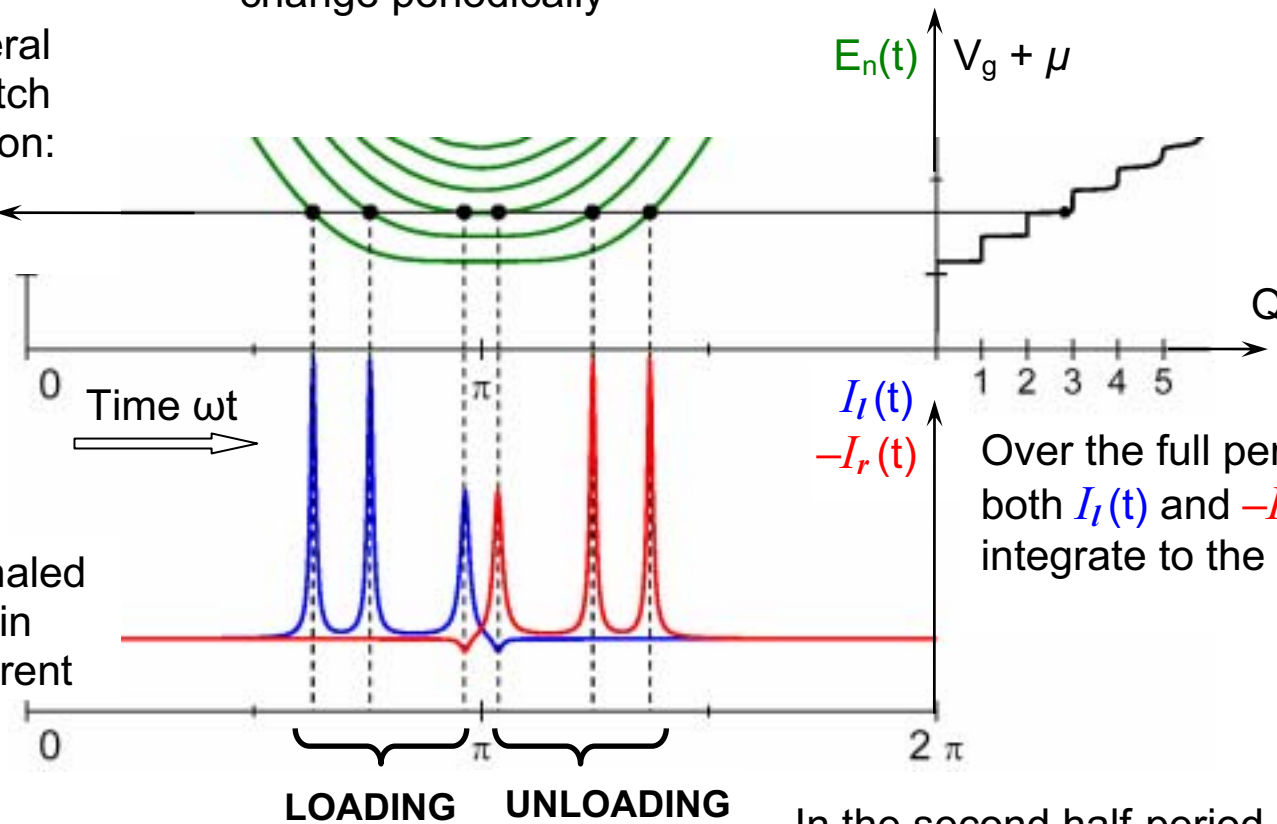
How does the staircase form?

Eigenenergies $E_n(t)$ of an isolated channel (no gates, no leads) change periodically

As time evolves, several energy levels can match the resonance condition:

$$E_n(t) = V_g + \mu$$

Each matching is signaled by a resonance peak in the instantaneous current



In the first half-period, the minimum of $V(x)$ is on the left $\rightarrow \Gamma_l \gg \Gamma_r$

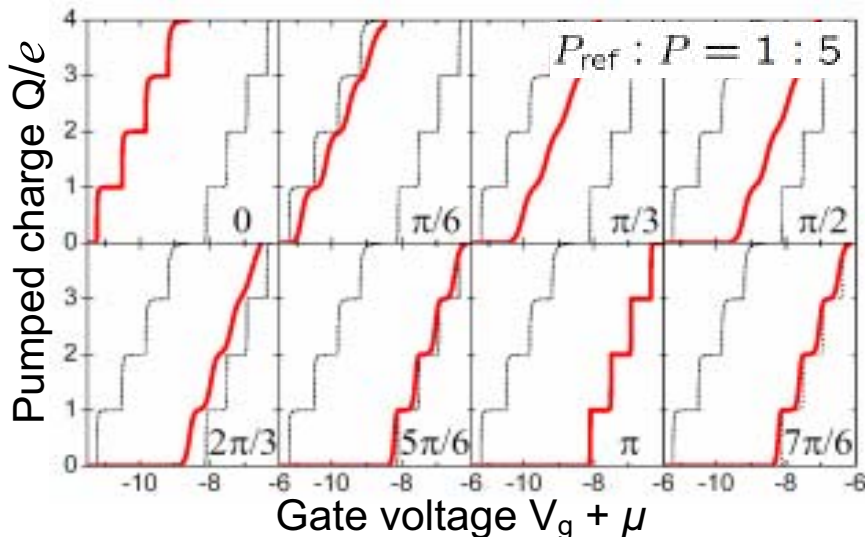
In the second half-period, the captured electrons are more likely to unload to the right, $\Gamma_l \ll \Gamma_r$

Resonance picture builds a bridge towards the 'moving QD' scenario

Secondary SAW effects



$$V(x, t) = -V_g + P \cos(qx - \omega t) + P_{\text{ref}} \cos(qx + \omega t + \varphi)$$



The sharpest steps are observed for $\varphi=0$ and π
Model parameters: $P = 8 \text{ J}$; $\lambda = 4 \text{ L}$; $L_s = \infty$; $N = 10$

- A weak counter-propagating wave due to reflection or a second SAW transducer
- Slope of the first step at $Q=e/2$ [6]

$$\frac{dQ}{dV_g} \approx \frac{A}{B + qL P_{\text{ref}} \sin \varphi} \quad (A, B = \text{const})$$

In a left-right symmetric channel $B = 0$

- Tuning the denominator to zero gives the sharpest staircase
- Dramatic increase in quantization accuracy for specifically tuned P_{ref} and φ has been observed experimentally [5]

What have we learned?

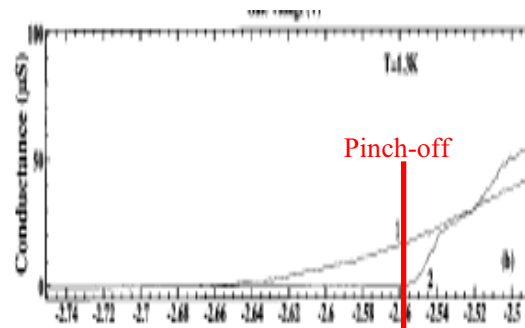
The SAW model example demonstrates a set of tools to study adiabatic quantum pumping due to complex potentials:

- Exact calculation reproduces the detailed qualitative features of the experiment
- Analysis of the model through the lens of the resonance approximation confirms the “moving quantum dot” scenario
- Two major obstacles on the way to reliable quantitative predictions: interactions and non-adiabaticity

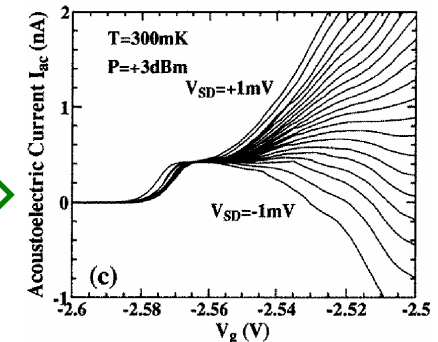
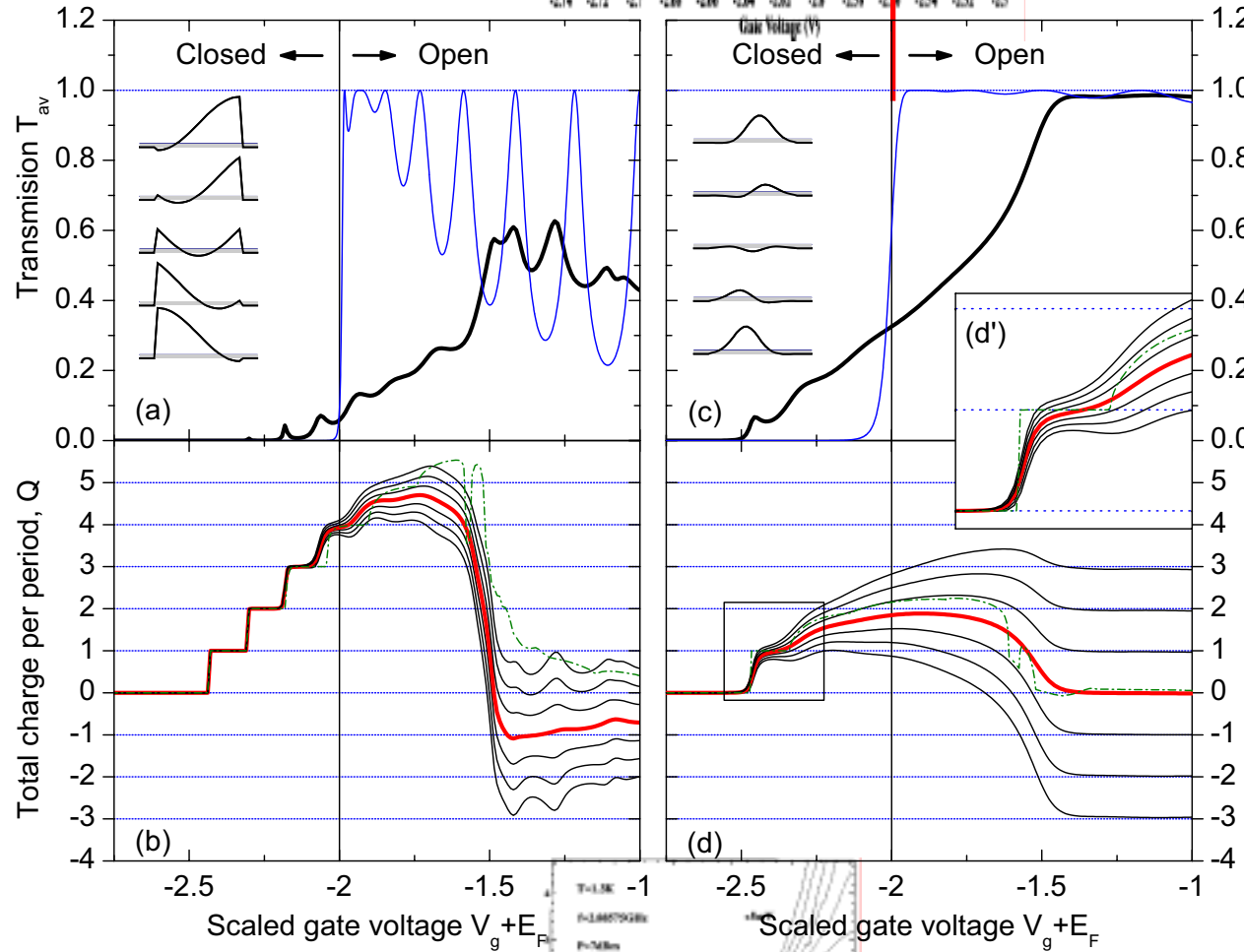
References

- [1] O. Entin-Wohlman, A. Aharony, and Y. Levinson, Phys. Rev. B **65**, 195411 (2002), cond-mat/0201073.
- [2] P. W. Brouwer, Phys. Rev. B **58**, 10 135 (1998).
- [3] A. Aharony and O. Entin-Wohlman, Phys. Rev. B **65**, 241401 (2002), cond-mat/0111053.
- [4] V. Kashcheyevs, A. Aharony, and O. Entin-Wohlman, Phys. Rev. B **69**, 195301 (2004), cond-mat/0308382.
- [5] J. Cunningham, V. I. Talyanskii, J. M. Shilton, M. Pepper, A. Kristensen, and P. E. Lindelof, Phys. Rev. B **62**, 1564 (2000), contains references to earlier experimental work.
- [6] V. Kashcheyevs, A. Aharony, and O. Entin-Wohlman, Eur. Phys. J B **39**, 385 (2004), cond-mat/0402590.
- [7] M. Moskalets and M. Büttiker, Phys. Rev. B **66**, 205320 (2002), cond-mat/0208356.

Talyanskii *et al.*,
PRB **56**, 15180 (1997)



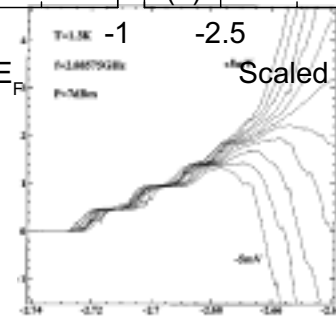
More details to discuss!



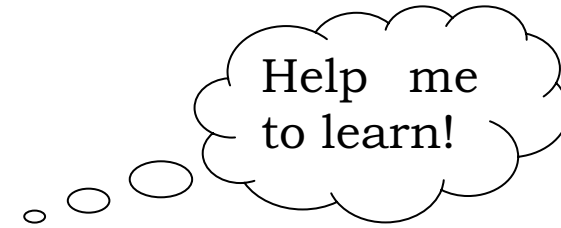
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- Smearing of conductance pinch-off
- Combination of source-drain bias and pumping
- Role of gradual screening

Talyanskii *et al.*,
PRB **56**, 15180 (1997)



Open questions



Interaction and correlation effects

For non-interacting electrons, we have a formalism to study the crossover from an open to a closed system. Can the resonance approximation be extended to include the **dynamic formation** of:

- Coulomb blockade?
- Kondo-type resonances?

The role of quantum interference

For $kT \gg \Gamma$, the interference in the leads can be ignored and the resonance approximation is equivalent to a rate equation approach [4, cond-mat version].

- What is truly quantum in “adiabatic quantum pumping”?
- How to define a classical limit?
- Are there any kind of “intensity” versus “interference” terms?

